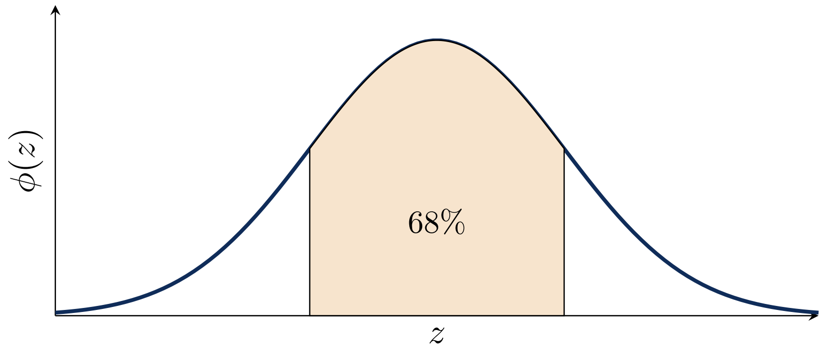
IB MATHEMATICS AI SL

TOPIC 4: STATISTICS & PROBABILITY

15 Long-Response Questions | Graded Medium to Very Hard



Overview & Syllabus Targets

- **Section A: Exam Questions** — 15 structured long-response problems graded in difficulty: Medium (Q1–Q5), Hard (Q6–Q11), and Very Hard (Q12–Q15). Each question begins on a new page.
- **Section B: Worked Solutions** — Step-by-step solutions with GDC keystrokes, parameter entry lists, exact marks, and custom Examiner Tips.
- **Syllabus Coverage** — Descriptive statistics, cumulative frequencies, Chi-Square independence and goodness-of-fit, Pearson's & Spearman's correlation, unpooled two-sample t-tests, Venn & tree probability, Binomial, and Normal distributions.

SECTION A: EXAM QUESTIONS

Question 1: Grouped Data, Cumulative Frequencies & Outliers (Medium) [6 Marks]

The daily water consumption (in litres) of 70 households in a suburban community was surveyed. The grouped frequency distribution is recorded in the table below:

Water Consumption, x (litres)	**Number of Households (Frequency)**
$0 \leq x < 10$	5
$10 \leq x < 20$	12
$20 \leq x < 30$	25
$30 \leq x < 40$	18
$40 \leq x < 50$	10

1. Write down the mid-interval value for the class $20 \leq x < 30$. **[1 mark]**
2. Use your GDC to estimate the mean and the sample standard deviation of this water consumption dataset. **[2 marks]**
3. An independent audit reveals that the lower quartile (Q_1) of this distribution is 19.5 litres and the upper quartile (Q_3) is 35.2 litres.
   (a) Calculate the Interquartile Range (IQR). **[1 mark]**
   (b) Determine the boundaries for identifying outliers. Hence, state whether a household consuming 60 litres is classified as an outlier. **[2 marks]**

Question 2: Chi-Square Test for Independence (Medium) [6 Marks]

A fitness center manager conducts a survey to determine whether the preferred choice of group exercise programs is independent of a participant's age group. A random sample of 300 members yields the following contingency table:

	Pilates	**Yoga**	**CrossFit**
Under 35 years	20	30	50
35 to 55 years	40	40	20
Over 55 years	50	30	20

A Chi-Square test for independence is performed at the 5% level of significance.

1. State the null and alternative hypotheses for this test. **[2 marks]**
2. Show that the expected frequency of members over 55 years choosing Pilates is approximately 36.7. **[2 marks]**
3. Using your GDC, write down:
   (a) the degrees of freedom; **[1 mark]**
   (b) the p-value for the test. **[1 mark]**
4. State the statistical conclusion of this test. Justify your answer. **[1 mark]**

Question 3: Pearson's Correlation & Linear Forecasting Limits (Medium) [6 Marks]

A sales director records the advertising expenditure, x (in thousands of dollars), and the resulting quarterly revenue, y (in thousands of dollars), for 8 regional branch outlets:

Expenditure, x	2.0	3.0	4.5	5.0	6.2	7.0	8.5	10.0
Revenue, y	45	56	68	75	85	92	110	124

1. Write down the Pearson product-moment correlation coefficient, r, for this data. Describe the strength and direction of the linear relationship. **[2 marks]**
2. Find the equation of the regression line of y on x in the form $y = ax + b$. **[2 marks]**
3. Use your regression model to:
   (a) predict the quarterly revenue for an outlet that spends \$5 500 on advertising; **[1 mark]**
   (b) explain why using this model to predict the revenue for an outlet spending \$25 000 might be unreliable. **[1 mark]**

Question 4: Venn Diagrams & Probability Rules (Medium) [6 Marks]

For two events A and B, it is known that the probability of A occurring is $P(A) = 0.60$, the probability of B occurring is $P(B) = 0.45$, and the probability of at least one of the events occurring is $P(A \cup B) = 0.78$.

1. Calculate the intersection probability, $P(A \cap B)$. **[2 marks]**
2. Draw a fully labeled Venn diagram displaying the probabilities for each of the four separate regions. **[2 marks]**
3. Show whether the events A and B are:
   (a) independent; **[1 mark]**
   (b) mutually exclusive. **[1 mark]**

Question 5: Normal Distribution Quality Inspection Boundaries (Medium) [6 Marks]

The weight of structural steel bearings produced at a manufacturing facility is normally distributed with a mean of 250 g and a standard deviation of 8 g.

1. Find the probability that a randomly selected bearing weighs between 242 g and 258 g. **[2 marks]**
2. Quality control rejects any steel bearings that weigh less than 235 g. Calculate the percentage of bearings that are expected to be rejected. **[2 marks]**
3. The heaviest 5% of the bearings are classified as "heavy grade" and are redirected to a specialized construction line. Find the minimum weight threshold required for a bearing to be classified as heavy grade. **[2 marks]**

Question 6: Spearman's Rank vs. Pearson's Correlation (Hard) [8 Marks]

An agricultural research laboratory measures the growth response of a specific weed species under varying applications of a bio-stimulant. The dosage, x (milligrams), and corresponding weed height, y (centimetres), are recorded:

Dosage, x (mg)	1.0	2.0	3.0	4.0	5.0	6.0	7.0
Height, y (cm)	10	12	15	25	60	200	1000

1. Copy and complete the table of ranks for x and y. **[2 marks]**
2. Write down:
   (a) the Spearman's rank correlation coefficient, r_s; **[1 mark]**
   (b) the Pearson product-moment correlation coefficient, r. **[1 mark]**
3. Explain the mathematical reason why there is a notable discrepancy between the values of r and r_s for this dataset. **[2 marks]**
4. State, with a reason, which correlation coefficient provides a more appropriate description of the relationship between bio-stimulant dosage and weed growth. **[2 marks]**

Question 7: Chi-Square Goodness-of-Fit Genetic Models (Hard) [8 Marks]

In a classical genetic inheritance experiment, a biologist crosses two hybrid pea plants. According to Mendelian genetics, the four resulting offspring seed phenotypes are expected to appear in a ratio of $9 : 3 : 3 : 1$.
In a sample of 160 offspring seeds, the observed frequencies are recorded:

Phenotype Category	**Round Yellow**	**Round Green**	**Wrinkled Yellow**	**Wrinkled Green**
Observed Frequency	84	35	29	12

A Chi-Square goodness-of-fit test is conducted at the 5% significance level to determine if the experimental results conform to the theoretical Mendelian ratio.

1. Calculate the expected frequencies for each of the four phenotype categories under the genetic model. **[2 marks]**
2. State the null and alternative hypotheses for this test. **[2 marks]**
3. Calculate the Chi-Square test statistic (χ^2_{calc}) showing your working. **[2 marks]**
4. Using your GDC, find the p-value. State whether the experimental data supports the Mendelian genetic inheritance model. **[2 marks]**

Question 8: Two-Sample t-Test for Educational Methods (Hard) [8 Marks]

A school district is evaluating two distinct teaching methods, Method A and Method B, for a high school chemistry course. Two separate, independent classes of students are taught using the different methods, and their final examination percentages are recorded:

- **Method A (12 students):** $70, 72, 75, 78, 80, 82, 68, 74, 76, 79, 81, 71$
- **Method B (15 students):** $78, 82, 85, 80, 88, 90, 75, 83, 81, 84, 86, 79, 87, 89, 76$

A two-sample t-test is performed at the 5% significance level to test if the mean exam score of students taught by Method B is higher than those taught by Method A. It is not assumed that the population variances are equal.

1. State the null and alternative hypotheses. **[2 marks]**
2. Write down the mean and the sample standard deviation for both groups. **[2 marks]**
3. Using your GDC, write down:
   (a) the t-statistic; **[1 mark]**
   (b) the p-value. **[1 mark]**
4. Interpret the result of the t-test in the context of the school district's decision. **[2 marks]**

Question 9: Binomial Distribution & Quality Defect Batches (Hard) [8 Marks]

An automated electronics assembly line produces smart-sensor microchips. Historically, the defect rate of these chips is constant at 8%. The chips are packaged in batches of 15 for shipping.

1. Find the probability that a randomly selected batch contains:
   (a) exactly 2 defective microchips; **[2 marks]**
   (b) at least 3 defective microchips. **[2 marks]**
2. Calculate the expected number of defective microchips in a batch of 15. **[1 mark]**
3. Calculate the standard deviation of the number of defective microchips in a batch. **[1 mark]**
4. A shipping crate contains 120 individual batches. Find the expected number of batches in a crate that contain no defective microchips. **[2 marks]**

Question 10: Medical Diagnostics Bayes' Theorem Tree Modeling (Hard) [8 Marks]

A medical diagnostic lab implements a screening test for a rare biological marker that is present in 1.5% of the general population.

- If the marker is present (M), the test correctly returns a positive result with a probability of 0.96 (true positive).
- If the marker is absent (M'), the test incorrectly returns a positive result with a probability of 0.04 (false positive).

1. Draw a fully labeled probability tree diagram representing this diagnostic system, showing all algebraic and numerical path probabilities. **[3 marks]**
2. Calculate the probability that a randomly selected individual from the population tests positive. **[2 marks]**
3. A patient receives a positive test result. Calculate the probability that they actually possess the rare biological marker. **[3 marks]**

Question 11: Normal Distribution with Dual Unknown Parameters (Hard) [8 Marks]

The volume of beverage dispensed by an automated bottling machine is modeled by a normal distribution with mean μ and standard deviation σ.
Measurements of a large test run indicate that:

- 15% of the bottles contain less than 40 ml of the beverage.
- 8% of the bottles contain more than 80 ml of the beverage.

1. Write down two simultaneous linear equations in terms of μ and σ by translating the probability statements into standard normal z-scores. **[4 marks]**
2. Solve the system of equations to determine the value of the mean, μ, and the standard deviation, σ, correct to three significant figures. **[3 marks]**
3. Under this bottling setup, find the probability that a bottle contains more than 50 ml. **[1 mark]**

Question 12: Asymmetric 3-Set Venn Diagrams & Conditional Constraints (Very Hard) [9 Marks]

A comprehensive survey of 120 university science students tracks their enrollment in three laboratory electives: Biology (B), Chemistry (C), and Physics (P). The following enrollment figures are recorded:

- 65 students take Biology.
- 55 students take Chemistry.
- 40 students take Physics.
- 25 students take Biology and Chemistry.
- 18 students take Biology and Physics.
- 15 students take Chemistry and Physics.
- x students take all three electives.

1. Represent this information in a Venn diagram in terms of x. **[3 marks]**
2. Show that the number of students who take none of the three electives is represented by the algebraic expression $18 - x$. **[2 marks]**
3. Given that exactly 10 students take none of the three electives, find the value of x. **[1 mark]**
4. Using $x = 8$, calculate the probability that a randomly chosen student takes:
   (a) Biology, given that they take at least two of the electives; **[2 marks]**
   (b) Chemistry, given that they do not take Physics. **[1 mark]**

Question 13: Normal Thresholds Linked to Binomial Sampling (Very Hard) [9 Marks]

The mass of premium organic apples harvested at an orchard is normally distributed with a mean of 150 g and a standard deviation of 12 g.

1. Apples are sorted into three commercial grades:

   - **Small:** Mass less than 135 g
   - **Large:** Mass more than 170 g
   - **Medium:** Mass between 135 g and 170 g

   Find the probability that a randomly chosen harvested apple is graded as:

   (a) Small; **[1 mark]**

   (b) Large; **[1 mark]**

   (c) Medium. **[1 mark]**

A random sample of 20 harvested apples is selected.

2. Find the probability that:

   (a) exactly 3 apples are classified as Small; **[2 marks]**

   (b) at most 2 apples are classified as Large; **[2 marks]**

   (c) at least 15 apples are classified as Medium. **[2 marks]**

Question 14: Unpooled t-Tests & Satisfaction Chi-Square Integration (Very Hard) [9 Marks]

A sports science institute evaluates two training regimes, Regime X and Regime Y, designed to reduce resting heart rate. The drops in resting heart rate (in beats per minute) are recorded for two independent participant groups:

- **Regime X (15 participants):** Mean heart rate drop $\bar{x}_1 = 12.4$ bpm, standard deviation $s_1 = 3.2$ bpm.
- **Regime Y (18 participants):** Mean heart rate drop $\bar{x}_2 = 14.8$ bpm, standard deviation $s_2 = 4.1$ bpm.

1. Perform a two-sample t-test (unpooled) at the 5% significance level to determine if Regime Y leads to a higher mean heart rate drop than Regime X. Write down your hypotheses, calculated t-statistic, p-value, and final conclusion. **[4 marks]**

In addition, the participants' satisfaction with their assigned regime was classified:

	High Satisfaction	**Medium Satisfaction**	**Low Satisfaction**
Regime X	8	5	2
Regime Y	6	8	4

2. Conduct a Chi-Square test for independence on this satisfaction data at the 5% significance level.
   (a) Write down the table of expected frequencies. **[2 marks]**
   (b) State the calculated χ^2 value, the degrees of freedom, and the corresponding p-value. **[2 marks]**
   (c) State the final decision regarding the independence of satisfaction and training regime. **[1 mark]**

Question 15: Influential Outliers & Leverage in Bivariate Regressions (Very Hard) [10 Marks]

An educational psychologist tracks the number of weekly hours spent studying mathematics, x, and the final exam score, y, achieved by a class of 9 students. The data is recorded as follows:

Hours, x	2.0	4.0	6.0	8.0	10.0	12.0	14.0	16.0	25.0
Exam Score, y	45	52	58	65	70	78	83	89	40

1. For the full dataset of 9 students:
   (a) write down the Pearson correlation coefficient, r_1; **[1 mark]**
   (b) find the equation of the regression line of y on x in the form $y = a_1x + b_1$. **[2 marks]**
2. The data point $(25.0, 40)$ is identified as an outlier representing a student who studied extensively but suffered from severe performance anxiety during the exam.
   Remove this outlier and re-evaluate the remaining 8 data points:
   (a) write down the revised Pearson correlation coefficient, r_2; **[1 mark]**
   (b) find the revised equation of the regression line of y on x in the form $y = a_2x + b_2$. **[2 marks]**
3. Compare your results from parts (1) and (2).
   (a) Describe the dramatic effect the single outlier has on the correlation strength and the regression slope. **[2 marks]**
   (b) Discuss the ethical and analytical implications of deciding whether to include or exclude such outliers when publishing research models. **[2 marks]**

SECTION B: WORKED SOLUTIONS

Worked Solution & Mark Distribution - Question 1

1. **Mid-Interval Value:**
   The mid-interval of the class interval $20 \leq x < 30$:

$$\text{Midpoint} = \frac{20+30}{2} = 25 \text{ litres}\textbf{A1}$$

2. **Estimated Mean and Sample Standard Deviation:**
   Enter the mid-interval values ($L_1 =$) and frequencies ($L_2 =$) into the GDC's 1-Variable Statistics:

$$\text{Mean } \bar{x} = 27.3 \text{ litres (to 3 s.f.)}\textbf{A1}$$

$$\text{Sample Standard Deviation } s_x = 11.2 \text{ litres (to 3 s.f.)}\textbf{A1}$$

   Note: Award method mark if population standard deviation $\sigma_x \approx 11.1$ is stated instead but penalized in final accuracy if not specified as sample statistic s_x.

3. **Outlier Analysis:**

   (a) Interquartile Range:

$$\text{IQR} = Q_3 - Q_1 = 35.2 - 19.5 = 15.7 \text{ litres}\textbf{A1}$$

   (b) Outlier Boundaries:

$$\text{Lower Boundary} = Q_1 - 1.5 \times \text{IQR} = 19.5 - 1.5(15.7) = 19.5 - 23.55 = -4.05 \text{ litres}\textbf{(M1)}$$

$$\text{Upper Boundary} = Q_3 + 1.5 \times \text{IQR} = 35.2 + 1.5(15.7) = 35.2 + 23.55 = 58.75 \text{ litres}\textbf{(A1)}$$

   Since 60 litres $>$ 58.75 litres, the household water consumption of 60 litres lies strictly beyond the upper boundary and is classified as an **outlier**. **A1**

Examiner Tip & Common Trap

When calculating statistics for grouped frequency tables on your GDC, always ensure you have listed the frequency list parameter in your setup. Leaving the frequency setting as a default value of "1" instead of pointing to your frequency list is a common error.

Worked Solution & Mark Distribution - Question 2

1. **Hypotheses Formulation:**
   H_0: The preferred choice of group exercise programs is independent of a participant's age group. **A1**
   H_1: The preferred choice of group exercise programs is dependent on a participant's age group. **A1**

2. **Expected Frequency Calculation:**
   Total number of participants over 55 years $= 50 + 30 + 20 = 100$. **(A1)**
   Total number of participants choosing Pilates $= 20 + 40 + 50 = 110$.
   Grand Total $N = 300$.

$$E_{3,1} = \frac{\text{Row Total} \times \text{Column Total}}{\text{Grand Total}} = \frac{100 \times 110}{300} = \frac{11\,000}{300} = 36.666... \approx 36.7\textbf{M1A1AG}$$

3. **GDC Results:**
   Input the 3x3 table into GDC Matrix $[A]$ and run the χ^2 2-Way Test:
   (a) Degrees of freedom: $df = (\text{rows} - 1) \times (\text{cols} - 1) = (3 - 1) \times (3 - 1) = 4$. **A1**
   (b) Calculated p-value: $p = 5.28 \times 10^{-7}$ (or 0.000000528). **A1**

4. **Statistical Conclusion:**
   Since the p-value (5.28×10^{-7}) is strictly less than the level of significance $\alpha = 0.05$: We **reject H_0** (or accept H_1). There is highly significant evidence to suggest that exercise choice is dependent on age. **R1**

Examiner Tip & Common Trap

Hypothesis statements must be written in terms of "independence" and "dependence". Never use the term "correlation" or "linear relationship" when describing Chi-Square test hypotheses.

Worked Solution & Mark Distribution - Question 3

1. **Pearson's Correlation Coefficient (r):**
Enter x and y into GDC lists L_1 and L_2 and perform Linear Regression:

$$r = 0.997\textbf{A1}$$

This value represents an **extremely strong positive linear correlation** between advertising expenditure and quarterly revenue. **A1**

2. **Regression Equation of y on x:**
From GDC output:

$$a = 9.84507... \quad \text{and} \quad b = 25.4330...$$

The regression line equation is:

$$y = 9.85x + 25.4\textbf{A1A1}$$

3. **Forecasting Analysis:**

   (a) Substitute $x = 5.5$ (since \$5 500 is 5.5 thousand dollars) into the regression line:

   $$y = 9.84507(5.5) + 25.4330 = 54.1478 + 25.4330 = 79.580... \implies \$79\,600\textbf{A1}$$

   (b) Spending \$25 000 represents an input of $x = 25.0$. This is far outside the experimental domain of the observed data ($2.0 \leq x \leq 10.0$). This process is called **extrapolation** and the linear model's behavior is highly unreliable at this range. **R1**

Examiner Tip & Common Trap

Check the scale units of the variables before substitution. Since x is defined in "thousands of dollars", a spending of \$5 500 must be input as $x = 5.5$, not 5500.

Worked Solution & Mark Distribution - Question 4

1. **Intersection Probability Calculation:**
   Using the general probability addition rule:

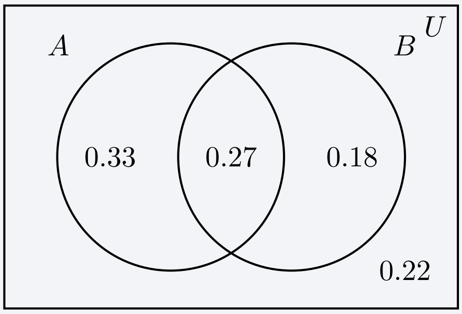
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)\textbf{(M1)}$$

   Substitute the known probabilities:

$$0.78 = 0.60 + 0.45 - P(A \cap B)\textbf{(A1)}$$

$$P(A \cap B) = 1.05 - 0.78 = 0.27\textbf{A1}$$

2. **Venn Diagram Representation:**



   Marks: Award A1 for three correct circle regions $(0.33, 0.27, 0.18)$ *and A1 for the correct exterior probability* (0.22) *inside the box.*

3. **Property Verification:**
   (a) Independence: Check if $P(A) \times P(B) = P(A \cap B)$:

$$P(A) \times P(B) = 0.60 \times 0.45 = 0.27$$

   Since $0.27 = P(A \cap B)$, the events are **independent**. **A1**

   (b) Mutual Exclusivity: Since $P(A \cap B) = 0.27 \neq 0$, the events **can occur simultaneously** and are **not mutually exclusive**. **A1**

Examiner Tip & Common Trap

To show independence, you must display a clear arithmetic comparison. Stating "they are independent because they don't affect each other" without showing $P(A) \times P(B) = P(A \cap B)$ will result in a zero mark for reasoning.

Worked Solution & Mark Distribution - Question 5

1. **Probability between 242 g and 258 g:**
   Let $X \sim N(250, 8^2)$. Using GDC normal cumulative distribution function (`normalCDF`):

$$P(242 \leq X \leq 258) = \texttt{normalCDF}(242, 258, 250, 8)\textbf{(M1)}$$

$$P(242 \leq X \leq 258) = 0.682689... \implies 0.683\textbf{A1}$$

2. **Rejection Probability (Weight ¡ 235 g):**

$$P(X < 235) = \texttt{normalCDF}(-\infty, 235, 250, 8)\textbf{(M1)}$$

$$P(X < 235) = 0.030396... \implies 3.04\% \text{ of bearings are rejected.}\textbf{A1}$$

3. **Specialized Grade Weight Threshold (k):**
   We want to find k such that $P(X > k) = 0.05$, which is equivalent to $P(X < k) = 0.95$. **(M1)**
   Using GDC inverse normal function (`invNorm`):

$$k = \texttt{invNorm}(0.95, 250, 8)$$

$$k = 263.158... \implies 263 \text{ g}\textbf{A1}$$

Examiner Tip & Common Trap

When using the inverse normal distribution solver, the input area parameter is conventionally the cumulative area from the far-left to the threshold. If a question defines ”the heaviest 5%”, you must input an area parameter of 0.95.

Worked Solution & Mark Distribution - Question 6

1. **Rank Table Construction:**
   Since both lists are already arranged in strictly ascending order:

Rank x	1	2	3	4	5	6	7
Rank y	1	2	3	4	5	6	7

   Marks: Award A1 for Rank x and A1 for Rank y.

2. **Correlation Calculations:**

   (a) Since the ranks match up perfectly, Spearman's rank correlation coefficient is:

   $$r_s = 1.00\mathbf{A1}$$

   (b) Performing linear regression on the raw data pairs on GDC:

   $$r = 0.719\mathbf{A1}$$

3. **Reason for Discrepancy:**
   Pearson's correlation coefficient r measures the strength of a linear relationship and is heavily affected by the exponential style of growth seen between dosage and height. Spearman's rank r_s measures the strength of any monotonic (consistently increasing or decreasing) relationship regardless of its curvature. **R2**

4. **Better Descriptor Decision:**
   Spearman's rank ($r_s = 1.00$) is more appropriate. **A1**
   The biological relationship is perfectly monotonic—every increase in dosage leads to an increase in height—which is captured by r_s. The low Pearson value (0.719) incorrectly suggests a weak correlation simply because the trend is non-linear. **R1**

Examiner Tip & Common Trap

Spearman's rank correlation is highly robust against non-linear scaling and extreme values (outliers) because it replaces actual values with rank positions.

Worked Solution & Mark Distribution - Question 7

1. **Expected Frequencies Calculation:**
   Sum of ratio parts $= 9 + 3 + 3 + 1 = 16$.

$$E_1 = \frac{9}{16} \times 160 = 90\textbf{A1}$$
$$E_2 = E_3 = \frac{3}{16} \times 160 = 30\textbf{A1}$$
$$E_4 = \frac{1}{16} \times 160 = 10$$

2. **Goodness-of-Fit Hypotheses:**
   H_0: The offspring pea seed phenotypes conform to a $9 : 3 : 3 : 1$ ratio. **A1**
   H_1: The offspring pea seed phenotypes do not conform to a $9 : 3 : 3 : 1$ ratio. **A1**

3. **Calculate Chi-Square statistic:**
   Using $\chi^2_{\text{calc}} = \sum \frac{(O_i - E_i)^2}{E_i}$:

$$\chi^2_{\text{calc}} = \frac{(84-90)^2}{90} + \frac{(35-30)^2}{30} + \frac{(29-30)^2}{30} + \frac{(12-10)^2}{10}\textbf{(M1)}$$
$$\chi^2_{\text{calc}} = \frac{36}{90} + \frac{25}{30} + \frac{1}{30} + \frac{4}{10} = 0.40 + 0.8333 + 0.0333 + 0.40\textbf{(A1)}$$
$$\chi^2_{\text{calc}} = 1.6667 \approx 1.67\textbf{A1}$$

4. **p-value and Decision:**
   With degrees of freedom $df = 4 - 1 = 3$, running χ^2 GOF on GDC yields:

$$p = 0.644\textbf{A1}$$

   Since $p = 0.644 > 0.05$, we **fail to reject H_0**. The experimental data supports the Mendelian genetic inheritance model. **A1**

Examiner Tip & Common Trap

When calculating degrees of freedom for a Chi-Square goodness-of-fit test, it is $k - 1$ where k is the number of outcome categories. Do not use the formula $(\text{rows} - 1)(\text{cols} - 1)$, which only applies to contingency tables.

Worked Solution & Mark Distribution - Question 8

1. **Hypotheses Formulation:**
   H_0: $\mu_A = \mu_B$ (The mean final exam score is equal for both methods). **A1**
   H_1: $\mu_B > \mu_A$ (The mean score for Method B is higher than Method A). **A1**

2. **Sample Descriptive Statistics:**
   Input lists on GDC to calculate 1-Var stats for each:

   $$\text{Group A: } N_1 = 12, \quad \bar{x}_A = 75.5, \quad s_A = 4.60\textbf{A1}$$

   $$\text{Group B: } N_2 = 15, \quad \bar{x}_B = 82.9, \quad s_B = 4.69\textbf{A1}$$

3. **t-Test parameters from GDC:**
   Perform a **2-Sample t-Test** with settings: $\mu_1 < \mu_2$, Pooled: No.

   (a) Calculated t-statistic: $t = -4.10$ **A1**

   (b) Calculated p-value: $p = 0.00021$ **A1**

   Note: Award $p = 0.00041$ if two-tailed test was performed in error instead of one-tailed.

4. **School District Decision:**
   Since $p = 0.00021 < 0.05$, we **reject H_0** in favor of H_1. **R1**
   The district has sufficient evidence to conclude that teaching Method B results in a significantly higher mean exam score than Method A. **A1**

Examiner Tip & Common Trap

Identify whether a t-test is one-tailed or two-tailed. The phrase ”test if the mean score of Method B is *higher* than Method A” indicates a one-tailed test. Performing a two-tailed test will double your p-value.

Worked Solution & Mark Distribution - Question 9

1. **Binomial Probabilities for a Batch ($n = 15$, $p = 0.08$):**

   (a) Exact Probability $P(X = 2)$: Using binomial probability density function (`binomPDF`):

   $$P(X = 2) = \binom{15}{2}(0.08)^2(0.92)^{13} = 105(0.0064)(0.3382...)\textbf{(M1)}$$

   $$P(X = 2) = 0.22728... \implies 0.227\textbf{A1}$$

   (b) Cumulative Probability $P(X \geq 3)$:

   $$P(X \geq 3) = 1 - P(X \leq 2) = 1 - \texttt{binomCDF}(15, 0.08, 2)\textbf{(M1)}$$

   $$P(X \leq 2) = \texttt{binomCDF}(15, 0.08, 2) \approx 0.88047...$$

   $$P(X \geq 3) = 1 - 0.88047... = 0.1195... \implies 0.120\textbf{A1}$$

2. **Expected Value:**

   $$E(X) = n \times p = 15 \times 0.08 = 1.2 \text{ defective microchips}\textbf{A1}$$

3. **Standard Deviation:**

   $$\sigma = \sqrt{np(1-p)} = \sqrt{15(0.08)(0.92)} = \sqrt{1.104} \approx 1.05 \text{ microchips}\textbf{A1}$$

4. **Crate Expected Batches with No Defects:**
   The probability a single batch has no defects is $P(X = 0) = 0.92^{15} \approx 0.286297$. **M1**
   The expected number of such batches in a crate of 120:

   $$\text{Expected Value} = 120 \times 0.286297 = 34.35... \implies 34.4 \text{ batches}\textbf{A1}$$

Examiner Tip & Common Trap

When calculating "at least" probabilities with GDC cumulative functions, remember that GDC calculators evaluate $P(X \leq k)$. To calculate $P(X \geq 3)$, rewrite it as $1 - P(X \leq 2)$.

Worked Solution & Mark Distribution - Question 10

1. **Probability Tree Diagram:**

Branch	Primary	Then Positive	Then Negative
Absent (M')	0.985	0.04 (false positive)	0.96
Present (M)	0.015	0.96 (true positive)	0.04

Marks: Award A1 for primary branches ($0.015, 0.985$), A1 for present sub-branches ($0.96, 0.04$), and A1 for absent sub-branches ($0.04, 0.96$).

2. **Total Positive Test Probability:**
Using law of total probability:

$$P(+) = P(M \cap +) + P(M' \cap +)\textbf{(M1)}$$

$$P(+) = (0.015 \times 0.96) + (0.985 \times 0.04) = 0.0144 + 0.0394\textbf{(A1)}$$

$$P(+) = 0.0538 \text{ (or } 5.38\%)\textbf{A1}$$

3. **Conditional Probability $P(M \mid +)$:**
Using Bayes' formula:

$$P(M \mid +) = \frac{P(M \cap +)}{P(+)} = \frac{0.0144}{0.0538}\textbf{(M1)(A1)}$$

$$P(M \mid +) = 0.267657... \implies 0.268 \text{ (or } 26.8\%)\textbf{A1}$$

Examiner Tip & Common Trap

Even with a very high test accuracy (like 96%), if the disease is extremely rare in the general population, most positive results are false positives. Here, a patient testing positive has only a 26.8% chance of actually possessing the marker.

Worked Solution & Mark Distribution - Question 11

1. **System of Linear Equations (z-scores):**
   Using $z = \frac{x-\mu}{\sigma}$: For 15% less than 40 ml ($P(Z < z_1) = 0.15$):

$$z_1 = \texttt{invNorm}(0.15, 0, 1) = -1.03643...\textbf{A1}$$

$$\frac{40-\mu}{\sigma} = -1.03643 \implies \mu - 1.03643\sigma = 40\textbf{A1}$$

   For 8% more than 80 ml ($P(Z > z_2) = 0.08 \implies P(Z < z_2) = 0.92$):

$$z_2 = \texttt{invNorm}(0.92, 0, 1) = 1.40507...\textbf{A1}$$

$$\frac{80-\mu}{\sigma} = 1.40507 \implies \mu + 1.40507\sigma = 80\textbf{A1}$$

2. **Solving the System of Equations:**
   Subtract the first equation from the second equation to eliminate μ:

$$(\mu + 1.40507\sigma) - (\mu - 1.03643\sigma) = 80 - 40\textbf{(M1)}$$

$$2.44150\sigma = 40 \implies \sigma = \frac{40}{2.44150} \approx 16.383...\textbf{(A1)}$$

   Substitute $\sigma \approx 16.383$ back to find μ:

$$\mu = 40 + 1.03643(16.383) \approx 56.980...$$

   Thus, correct to 3 s.f.: $\mu = 57.0$ **ml** and $\sigma = 16.4$ **ml**. **A1**

3. **Probability of Volume ¿ 50 ml:**

$$P(X > 50) = \texttt{normalCDF}(50, \infty, 56.980, 16.383)\textbf{(M1)}$$

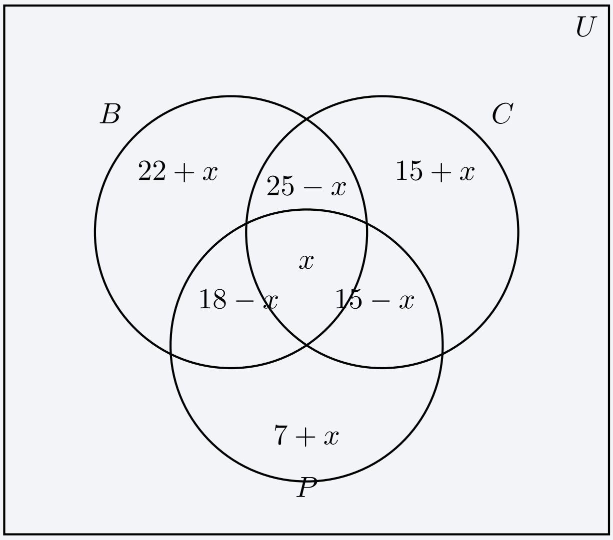
$$P(X > 50) = 0.66491... \implies 0.665\textbf{A1}$$

Examiner Tip & Common Trap

When calculating intermediate values of z-scores to solve simultaneous equations, preserve at least 5 decimal places of accuracy on your paper. Rounding too early to -1.04 and 1.41 will introduce errors in your final values of μ and σ.

Worked Solution & Mark Distribution - Question 12

1. **Venn Diagram Construction in terms of x:**



Marks: Award A3 if all interior regions are correctly displayed algebraically.

2. **Show none region is $18 - x$:**
Sum of all seven interior regions inside the circles:

$$S = (22 + x) + (15 + x) + (7 + x) + (25 - x) + (18 - x) + (15 - x) + x\textbf{(M1)}$$

Group and cancel variables:

$$S = 22 + 15 + 7 + 25 + 18 + 15 + (x + x + x - x - x - x + x)$$

$$S = 102 + x\textbf{(A1)}$$

Therefore, the number of students taking none of the three electives is:

$$\text{None} = 120 - (102 + x) = 18 - x\textbf{AG}$$

3. **Solving for x:**

$$18 - x = 10 \implies x = 8\textbf{A1}$$

4. **Ratios and Probabilities ($x = 8$):**

   (a) Let T be the set of students taking at least two electives. Number of students taking at least two electives $= (25 - 8) + (18 - 8) + (15 - 8) + 8 = 17 + 10 + 7 + 8 = 42$. **(A1)**
   Of those, the number taking Biology is $= 17 + 10 + 8 = 35$. **(A1)**

$$P(B \mid \geq 2) = \frac{35}{42} = \frac{5}{6} \approx 0.833\textbf{A1}$$

   (b) Given they do not take Physics (P'), $N(P') = 120 - 40 = 80$. **(A1)**
   Of those, those taking Chemistry are in regions Only C$+(B \cap C \cap P') = 23 + 17 = 40$.

$$P(C \mid P') = \frac{40}{80} = 0.50\textbf{A1}$$

Examiner Tip & Common Trap

For 3-set Venn diagrams with an unknown central intersection x, first write down the double-intersection regions as $(A \cap B) - x$. This simplifies the algebraic consolidation for the single-set regions.

Worked Solution & Mark Distribution - Question 13

1. **Normal Threshold Probabilities ($X \sim N(150, 12^2)$):**

   (a) Small grade ($X < 135$ g):

   $$P(X < 135) = \texttt{normalCDF}(-\infty, 135, 150, 12) = 0.1056... \implies 0.106\textbf{A1}$$

   (b) Large grade ($X > 170$ g):

   $$P(X > 170) = \texttt{normalCDF}(170, \infty, 150, 12) = 0.04779... \implies 0.0478\textbf{A1}$$

   (c) Medium grade ($135 \leq X \leq 170$ g):

   $$P(135 \leq X \leq 170) = 1 - (0.10565 + 0.04779) = 0.8465... \implies 0.847\textbf{A1}$$

2. **Binomial Sample Probabilities ($n = 20$):**

   (a) Exactly 3 Small ($p_s = 0.10565$):

   $$P(Y = 3) = \texttt{binomPDF}(20, 0.10565, 3)\textbf{(M1)}$$

   $$P(Y = 3) = 0.2014... \implies 0.201\textbf{A1}$$

   (b) At most 2 Large ($p_l = 0.04779$):

   $$P(Z \leq 2) = \texttt{binomCDF}(20, 0.04779, 2)\textbf{(M1)}$$

   $$P(Z \leq 2) = 0.9322... \implies 0.932\textbf{A1}$$

   (c) At least 15 Medium ($p_m = 0.84656$):

   $$P(W \geq 15) = 1 - P(W \leq 14) = 1 - \texttt{binomCDF}(20, 0.84656, 14)\textbf{(M1)}$$

   $$P(W \geq 15) = 1 - 0.0737... = 0.9262... \implies 0.926\textbf{A1}$$

Examiner Tip & Common Trap

This is a high-yield integrated probability problem. You must first use the normal distribution to calculate the probabilities of the subgroups. These outputs then become the binomial parameters (p) for the second section of the question.

Worked Solution & Mark Distribution - Question 14

1. **Two-Sample t-Test:**
   H_0: $\mu_X = \mu_Y$ (The mean drop is equal for both training regimes). **A1**
   H_1: $\mu_Y > \mu_X$ (Regime Y results in a higher mean drop). **A1**
   Using GDC unpooled 2-Sample t-test:

   $$t = -1.89\textbf{A1}$$

   $$p = 0.0343\textbf{A1}$$

   Note: The p-value for a one-tailed test is half of the two-tailed output: $0.0685/2 = 0.0343$.
   Since $p = 0.0343 < 0.05$, we **reject H_0**. There is sufficient evidence to conclude that Regime Y results in a higher mean drop. **A1**

2. **Chi-Square Test for Independence:**

   (a) Expected Frequencies table: Row Totals: Regime X = 15, Regime Y = 18. Col Totals: High = 14, Med = 13, Low = 6.

   $$E_{ij} = \frac{\text{Row Total} \times \text{Col Total}}{\text{Grand Total (33)}}$$

	High	**Medium**	**Low**
Regime X	6.36	5.91	2.73
Regime Y	7.64	7.09	3.27

   Marks: Award A2 if all entries are correct to 3 s.f.; A1 if some are incorrect.

   (b) GDC Test Parameters:

   $$\chi^2 = 1.38\textbf{A1}$$

   $$df = (2-1) \times (3-1) = 2\textbf{A1}$$

   $$p = 0.501\textbf{A1}$$

   (c) Statistical decision: Since $p = 0.501 > 0.05$, we **fail to reject H_0**. Satisfaction is independent of the training regime. **A1**

Examiner Tip & Common Trap

When working with summary statistics (means and standard deviations) instead of raw datasets, use the summary input screen of your GDC's 2-Sample t-test solver.

Worked Solution & Mark Distribution - Question 15

1. **Full Dataset (9 students):**
   (a) Pearson correlation coefficient: $r_1 = 0.143$ **A1**
   (b) Regression line: $y = 0.346x + 60.7$ **A2**
2. **Excluding the Outlier (8 students):**
   (a) Pearson correlation coefficient: $r_2 = 0.999$ **A1**
   (b) Regression line: $y = 3.14x + 39.2$ **A2**
3. **Outlier Analysis and Leverage:**
   (a) The single outlier $(25.0, 40)$ completely destroys the linear correlation—reducing r from an extremely strong 0.999 to a very weak 0.143. It also reduces the regression line slope from 3.14 (strong improvement per hour) to a nearly flat 0.346. **R2**
   (b) Outliers with high horizontal leverage can heavily bias regression lines. Excluding outliers is acceptable if there is a clear non-statistical justification (e.g. medical performance anxiety). However, removing data points simply to make a model look better is unethical. All data exclusions must be declared. **R2**

Examiner Tip & Common Trap

A single coordinate point placed far to the right or left can act as a powerful anchor point in linear regression. Always inspect data plots visually before attempting linear modelling.